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**Question Paper Code : 51325**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to : Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/ Computer and Communication Engineering/Artificial Intelligence and Data Science/ Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that  $p \rightarrow q$  and  $\neg p \vee q$  are equivalent.
2. Translate the statement 'Every real number except zero has a multiplicative inverse' into the statement involving nested quantifiers.
3. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?
4. State the pigeonhole principle.
5. Define degree of a vertex in an undirected graph.
6. Define path.
7. Is the set of integers under ordinary multiplication a group?
8. Prove that for each element  $a$  in a group  $G$ , there is a unique element  $b$  in  $G$  such that  $ab = ba = e$ .
9. Show that the 'greater than or equal' relation ( $\geq$ ) is a partial ordering on the set of integers.
10. Is the poset  $(\mathbb{Z}^+, |)$  a lattice?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the principle disjunctive normal form of  $(P \rightarrow Q) \wedge (P \rightleftharpoons R)$ . (8)
- (ii) Show that  $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ . (8)

Or

- (b) (i) Prove that if  $n = ab$ , where  $a$  and  $b$  are positive integers, then  $a \leq \sqrt{n}$  or  $b \leq \sqrt{n}$ . (8)
- (ii) Show that  $(\forall x)[P(x) \vee Q(x)] \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$ . (8)
12. (a) (i) Show that if  $n$  is an integer greater than 1, then  $n$  can be written as the product of primes. (8)
- (ii) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (8)

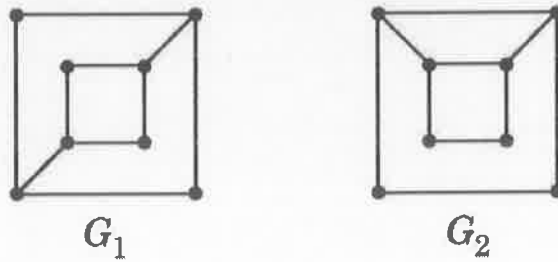
Or

- (b) (i) Find the number of 2-permutations with unlimited repetitions of  $\{a, b, c, d\}$ . (8)
- (ii) Solve  $s(n+2) - 5s(n+1) + 6s(n) = 0$  for  $n \geq 0$  with  $s(0) = 1$ ,  $s(1) = 1$ . (8)
13. (a) (i) Prove that a simple connected graph is bipartite if and only if it contains no odd cycle. (8)

- (ii) Draw the graph with the adjacency matrix  $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$  with respect to the ordering of  $a, b, c, d$ . (8)

Or

- (b) (i) Check whether the graphs are isomorphic or not? (8)



- (ii) Prove that, if  $G$  is a simple connected graph with  $n \geq s$  vertices and  $d(u) \geq \frac{n}{2}$ , for all  $u \in V(G)$ , then  $G$  is Hamiltonian graph. (8)

14. (a) (i) Prove that the subgroup of a cyclic group must be cyclic subgroup. (8)

- (ii) Prove that a subgroup  $H$  of  $G$  is normal in  $G$  if and only if  $xHx^{-1} \subseteq H$  for all  $x$  in  $G$ . (8)

Or

- (b) (i) State and prove Lagrange's theorem. (8)

- (ii) Prove that the Kernel of a homomorphism  $f$  from the group  $\langle G, * \rangle$  to a group  $\langle H, \Delta \rangle$  is a normal subgroup of the group  $\langle G, * \rangle$ . (8)

15. (a) (i) Draw the Hasse diagram representing the partial ordering  $\{(a, b) | a \text{ divides } b\}$  on  $\{1, 2, 3, 4, 6, 8, 12\}$ . (8)

- (ii) Prove that every chain is a distributive lattice. (8)

Or

- (b) (i) Find the values of the Boolean function represented by  $F(x, y, z) = xy + \bar{z}$ . (8)

- (ii) Prove that the De Morgan laws are valid in a Boolean Algebra. (8)